SUPPLEMENTARY WRITTEN SUBMISSION FROM DR. J. R. CUTHBERT

At yesterday’s meeting of the Finance Committee, one of the points which came up in discussion was the way in which relative population growth interacted with the Barnett formula to affect per-capita spending levels. I undertook to send to the Committee a copy of a paper on this which I had published in the Fraser of Allander Quarterly Economic Commentary. A copy of this paper can be found as the Annex to this note. However, it might be useful for the Committee if I summarise some of the main points, as follows.

Let $\theta$ denote the rate of growth in nominal public expenditure in England: (so 2% expenditure growth is equivalent to $\theta = 1.02$).

Let $\lambda$ denote the relative rate of population growth in England relative to Scotland: (so $\lambda = 1.002$ implies population growth in England is 0.2% faster than in Scotland.)

What the paper shows is that, (apart from a relatively unimportant term which depends on the lag in applying the population figures in Barnett), the behaviour of per-capita spending levels between Scotland and England depends critically on the relative values of $\theta$ and $\lambda$. There are two important cases to consider.

a) If, as was the case prior to the 2008 crash, nominal expenditure growth is greater than relative population growth, (that is, if $\theta$ is greater than $\lambda$), then per capita spending levels tend through time towards a limiting value, which is, effectively, given by the expression

$$\left(\frac{\theta - 1}{\theta - \lambda}\right)$$

If $\lambda = 1$, then formula (1) implies that the limiting value is 1. This is what was conventionally assumed to be the standard case, under which Barnett would lead in due course to equality of per capita spending levels between Scotland and England.

However, if $\lambda$ is greater than 1, which is, in fact, what has been the case historically, then the limiting value at formula (1) will be greater than 1: and can be very much greater than 1: as the following table illustrates.

<table>
<thead>
<tr>
<th>Values of the formula (1) expression</th>
<th>Expenditure growth (theta)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.01</td>
</tr>
<tr>
<td>Relative population growth (lambda)</td>
<td>1.00</td>
</tr>
<tr>
<td>1.001</td>
<td>1.11</td>
</tr>
<tr>
<td>1.002</td>
<td>1.25</td>
</tr>
<tr>
<td>1.003</td>
<td>1.43</td>
</tr>
<tr>
<td>1.004</td>
<td>1.67</td>
</tr>
<tr>
<td>1.005</td>
<td>2.00</td>
</tr>
</tbody>
</table>

b) The formula in para (7) of the Annex implies that, when the rate of expenditure growth is less than relative population growth, per capita spending levels in Scotland will increase relative to those in England, and will keep on doing so, (albeit, usually fairly slowly). This is the situation we have been in during the post-2008 era of
austerity: which has meant that the Barnett formula has provided a (relative) bonus to Scotland.

It is worth making one other point. The complexity of the results in this particular, (relatively simple), case is a specific illustration of a point which came up in another context in yesterday’s discussion at the Finance Committee. Namely, that it is not possible to determine what the overall effect of a system will be, unless it has been properly modelled. (This point came up yesterday in relation to the interaction of the Barnett formula post-Smith with Holtham indexation.)
The Effect of Relative Population Growth on the Barnett Squeeze.

J.R. Cuthbert

1) This note is concerned with the Barnett squeeze as narrowly defined, (that is leaving aside the impact of local sources of funding like non-domestic rates and council tax outwith the Scottish Executive’s DEL) and considers the implications of relative population growth in England compared to Scotland. It appears intuitively plausible that, if the population of England is growing relative to Scotland, then this will tend to dampen the rate of convergence of per capita spending levels in Scotland and England. It is difficult, however, without going into the algebra, to assess whether this statement is a truism which is likely to have little practical impact, or whether it is likely to have a significant impact on convergence rates in the real world.

2) To investigate this question further, an idealised model of how Barnett works has been set up. As will become apparent this model represents a simplified description of the public expenditure planning process and the application of the Barnett formula. Nevertheless this model captures the key aspects of the system, and enables some useful, and perhaps surprising, conclusions to be drawn.

3) The government plans its future programme of public expenditure on the basis of a three year rolling programme. Thus, in the Spending Review 2000 exercise (SR2000), the government set its programmes for the future years of 2001-02, 2002-03 and 2003-04. When the government comes to carry out its 2001 planning round, therefore, the first thing it has to do is to establish a starting, or baseline, figure for the new end year in the three year planning horizon. In this case this will be 2004-05. The way the government does this is of critical importance for the application of the Barnett formula. Since 1993, the government has set the new final year base line for each programme as being equal to the previous final year in cash: (source: personal communication from H.M.Treasury). Prior to 1993, the new final year base line was apparently set as the previous final year figure, uprated for inflation: (see H.M.Treasury (1997)). The government then considers what adjustments are required to the English baseline figures for each of the three future years. In the 2001 planning round it will be considering years 2002-03 to 2004-05. These adjustments to the English baseline figures will include any required uplift for inflation in the new final year. These adjustments then feed into the Barnett formula to give the required changes to the Scottish DEL for each of the three years. In applying the Barnett formula, Scotland is given its population share of changes to English department base lines, based on the most recently available historic population estimates for Scotland and England. Typically, therefore, the population shares used will relate to a date two years before the year in which the spending review is being carried out. Hence for the 2001 SR, the population estimates would relate to the year 1999. (For details on the operation of Barnett, see H.M.Treasury (2000)).

4) It will be apparent from the description in the previous paragraph why modelling the operation of Barnett is not simple. In particular, the Scottish DEL for any given year will be modified by the operation of Barnett in three successive
planning rounds, each time using population estimates relating to a different historic
date, and with a different lag each time between the date of the population estimates
and the year being adjusted. These lags will, typically, be five years, four years, and
three years successively. The average lag, therefore, between the date of the
population estimates used and the year being adjusted is likely to be of the order of
four years.

5) The following simplified model of how Barnett works abstracts from these
complications. In this model, the Scottish DEL for any given year is adjusted only
once by Barnett, when the new baseline for that year is first established. Another
simplification is the assumption that public expenditure in England is growing by a
constant percentage each year.

6) Specifically, the following notation and assumptions are used:-

- Let $E_t$ denote expenditure in England in year $t$, and $E^S_t$ expenditure in
Scotland.
- Let $p_t$ denote population in England in year $t$, and $p^S_t$ population in
Scotland.
- Let $R_t$ denote the ratio of per capita expenditures between Scotland and
England at time $t$.
- Let $k$ denote lag, (in years).

Suppose that

a) $E_{t+1} = \theta E_t$ : (i.e., expenditure in England grows at a constant rate.)

b) $\frac{p_{t+1}}{p_t} = \lambda \frac{p^S_{t+1}}{p^S_t}$ for all $t$, where $\lambda \geq 1$: (i.e., there is a constant relative
rate of growth of population in England relative to Scotland).

c) In the annual public expenditure planning round, the new final year baseline
is determined as being equal to the previous end year figure: and Barnett
applies only to that end year, with population shares determined at a lag $k$.

7) The above model can be solved, to show how the per capita spending relativity
between Scotland and England, $R_t$, evolves through time from its initial starting
value in year 0.
The relevant formula for $R_t$ is as follows:-

$$R_t = \left(\frac{\lambda}{\theta}\right)^{t} R_0 - \lambda^k \frac{(\theta - 1)}{(\theta - \lambda)} + \lambda^k \frac{(\theta - 1)}{(\theta - \lambda)} \frac{(\theta - 1)}{(\theta - \lambda)}, \text{ for } \lambda \neq 0.$$ (1)

The derivation of formula (1) is given in the Annex.

What formula (1) means is that, in the normal circumstance where $\frac{\lambda}{\theta} < 1$, then the
initial per capita spending relativity, $R_0$, will decay geometrically to the limiting
value $\lambda^k \frac{(\theta - 1)}{(\theta - \lambda)}$, which is a function of the expenditure growth rate in England,
the rate of relative population growth, and the lag.
8) If $\lambda = 1$, then the limiting value $\lambda^k \frac{(\theta - 1)}{(\theta - \lambda)}$ collapses to 1, (as is expected), and the rate of convergence is governed by the term $\theta^t$ : that is, the larger $\theta$ is, the faster the rate of convergence, which, again, is exactly as expected. However, if $1 < \lambda < \theta$, then the limiting value will be greater than 1: so the per capita spending ratio will never converge to unity.

9) In order to establish whether formula (1) just represents a curiosity, or whether it has any practical importance, it is necessary to know what range of values of $\lambda$ and $\theta$ is likely to be encountered in practice:-

a) The table below shows values of $\lambda$ back to 1961, for individual years, or averaged over groups of years, with projected values forward to 2021. The projected figures relate to the 1998 based official population projections: the figures on which the table is based are taken from the Annual Abstract of Statistics, 2000.

<table>
<thead>
<tr>
<th>Year</th>
<th>$\lambda$</th>
<th>Year</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1961</td>
<td></td>
<td>1996</td>
<td>1.0056</td>
</tr>
<tr>
<td>1971</td>
<td>1.0055</td>
<td>1997</td>
<td>1.0050</td>
</tr>
<tr>
<td>1981</td>
<td>1.0020</td>
<td>1998</td>
<td>1.0049</td>
</tr>
<tr>
<td>1991</td>
<td>1.0044</td>
<td>2001</td>
<td>1.0054</td>
</tr>
<tr>
<td>1992</td>
<td>1.0027</td>
<td>2006</td>
<td>1.0039</td>
</tr>
<tr>
<td>1993</td>
<td>1.0014</td>
<td>2011</td>
<td>1.0038</td>
</tr>
<tr>
<td>1994</td>
<td>1.0013</td>
<td>2021</td>
<td>1.0040</td>
</tr>
<tr>
<td>1995</td>
<td>1.0030</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For present purposes, it is relevant to note that, in the 1990s, $\lambda$ increased from a value of 1.0013 in 1994, to 1.0056 in 1996, and is projected to remain at a value around 1.005 until 2001.

b) As regards $\theta$, through the mid 1990s growth in government expenditure was typically in low single figures: for example, the annual growth in the aggregate control total for the expenditure of government departments was between 1.6% and 4% between 1993-94 and 1997-98. This contrasts with the current situation: for example, in SR2000, total DELs are projected to rise by 10.4%, 8.7%, 8.1%, and 7.1% over the next four years respectively.

10) For illustrative purposes, therefore, a $\lambda$ value of 1.005, and $\theta$ values of 1.03 and 1.09 are considered, (as being typical of values in the mid 1990s, and currently). An illustrative lag of 4 years is considered, as is a starting value of 1.2 for the level of public expenditure per head in Scotland relative to England, (close to the current value). The relevant results are illustrated in Charts 1 to 3.

Chart 1 shows the path of the per capita expenditure relativity, when $\theta=1.03$, and $\lambda=1$: in this case, of course, the limiting ratio is 1, and, because of the relatively small growth rate in expenditure in England, convergence is fairly slow.
Chart 1: Movement of Per Capita Exp Relative Towards Limit:
Theta=1.03, Lambda=1, lag=4

Chart 2 shows what happens when $\lambda = 1.005$ : (note that this is the only change from the Chart 1 case.) This time, a radically different picture emerges. The limiting value is now 1.22, (that is, above the starting value), so that what is observed is slow growth of $R_t$ to this level.

Finally, Chart 3 shows what happens when $\theta$ is increased to 1.09, but still retaining a $\lambda$ value of 1.005. In this case, the limiting value is 1.08. Although this value is higher than the limiting value of 1 in Chart 1, the values of $R_t$ in Chart 3 are actually below those in Chart 1 for about the first 20 years, because the rate of convergence to the limiting value is faster in Chart 3.
11) These results illustrate how levels of relative population growth similar to those currently being experienced can have a very marked effect indeed on the behaviour of relative per capita spending, when expenditure growth rates are in low single figures. However, this effect is significantly reduced when expenditure growth rates approach double figures. These findings are very relevant to the debate about the likely historical impact of the Barnett squeeze. At times during the 1990s, given the low growth rate in overall public expenditure, and the high relative growth rate of population in England which applied latterly, the model indicates that we would not have expected convergence in per capita spending levels. The situation now, with the high current levels of public expenditure growth, is markedly different.

12) Finally, it is worth noting that the illustrative results quoted here are not very sensitive to variations in the lag parameter, (which, it will be recalled, has been set at a value of 4 years, representing the likely average lag in practice.) This is illustrated by the following table, which shows the limiting value of the per capita expenditure ratio, given different choices of the lag parameter, and $\theta$, (and with $\lambda=1.005$ in each case).

<table>
<thead>
<tr>
<th>Lag (years)</th>
<th>1.03</th>
<th>1.06</th>
<th>1.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.206</td>
<td>1.096</td>
<td>1.064</td>
</tr>
<tr>
<td>2</td>
<td>1.212</td>
<td>1.102</td>
<td>1.069</td>
</tr>
<tr>
<td>3</td>
<td>1.218</td>
<td>1.107</td>
<td>1.075</td>
</tr>
<tr>
<td>4</td>
<td>1.224</td>
<td>1.113</td>
<td>1.08</td>
</tr>
</tbody>
</table>

In other words, the key factor driving the results illustrated in this note is the relative values of $\theta$ and $\lambda$.

REFERENCES

Notation and assumptions are as in paragraph 6.
From 6(a), it follows that \( E_t = \theta_t E_0 \). \hspace{1cm} (A1)
From 6(b), it follows that \( \frac{p^S_t}{p_t} = \frac{p^S_0}{p_0} \lambda^{-t} \). \hspace{1cm} (A2)
From 6(c), it follows that

\[
E^S_{t-1} = E^S_{t-1} + \frac{p^S_{t-k}}{p_{t-k}} (E_t - E_{t-1}) \\
= E^S_{t-1} + \frac{p^S_{t-k}}{p_{t-k}} \theta^{t-1}(\theta - 1)E_0 \hspace{1cm} \text{by (A1),}
\]

\[
= E^S_{t-1} + \lambda^{k-t} \theta^{t-1}(\theta - 1)E_0 \frac{p^S_0}{p_0} \hspace{1cm} \text{by (A2),}
\]

\[
= E^S_{t-1} + \lambda^{k-1}(\theta)\lambda^{-t}(\theta - 1)E_0 \frac{p^S_0}{p_0}.
\]

It follows, on summing the resulting geometric series in \( \frac{\theta}{\lambda} \), that

\[
E^S_{t-1} = E^S_0 + \lambda^{k-1}(\theta - 1)E_0 \frac{p^S_0(1 - (\frac{\theta}{\lambda})^{t-1})}{p_0(1 - \frac{\theta}{\lambda})}, \hspace{1cm} \text{when } \theta \neq \lambda, \hspace{1cm} (A3)
\]
and

\[
E^S_{t-1} = E^S_0 + t\lambda^{k-1}(\theta - 1)E_0 \frac{p^S_0}{p_0}, \hspace{1cm} \text{when } \theta = \lambda. \hspace{1cm} (A4)
\]

It follows from (A3), when \( \theta \neq \lambda \), that

\[
R_t = \frac{E^S_t p_t}{p^S_tE_t} = [E^S_0 + \lambda^{k-1}(\theta - 1)E_0 \frac{p^S_0(1 - (\frac{\theta}{\lambda})^{t-1})}{p_0(1 - \frac{\theta}{\lambda})}] \cdot \frac{\lambda^t p_0}{\theta^t E_0 p^S_0},
\]
which simplifies to

\[
R_t = \left(\frac{\lambda}{\theta}\right)^t [R_0 - \lambda^k \frac{(\theta - 1)}{(\theta - \lambda)}] + \lambda^k \frac{(\theta - 1)}{(\theta - \lambda)},
\]
which is the required expression.
It follows from (A4), when \( \theta = \lambda \), that,

\[
R_t = [E^S_0 + t\lambda^{k-1}(\theta - 1)E_0 \frac{p^S_0}{p_0}] \cdot \frac{p_0}{E_0 p^S_0}
\]
\[
= R_0 + t\lambda^{k-1}(\theta - 1),
\]
so in this special, (and unlikely), case, \( R_t \) grows linearly.